RBS

The Royal Bank of Scotland Group

IG: a new approach to pricing portfolio credit derivatives

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Portfolio credit derivatives

- In recent years, products with pay-offs depending on credit events from multiple reference entities have become popular.
- Such products are known as portfolio credit derivatives.
- Principal examples:
  - CDOs (collateralized debt obligations)
  - First to default baskets
  - CDO-squareds i.e. CDOs of CDOs

The pricing of portfolio credit derivatives

- Essentially a portfolio credit derivative pays cash-flows as a function of when the underlying assets default.
- Here we are concerned with derivatives that do not contain any optionality.
- So pricing is dependent upon modelling the joint default distribution of the underlying assets in a pricing measure.

Determining the joint default distribution

- Single default swaps are liquidly traded, and determine (with appropriate interpolations) the distribution of the default time of each name individually.
- The subtlety of pricing is therefore in determining the co-dependence of default times.
- There exists plenty of evidence that default times are not independent
  - default clustering in the past
  - the market price of portfolio credit derivatives is not consistent with independence

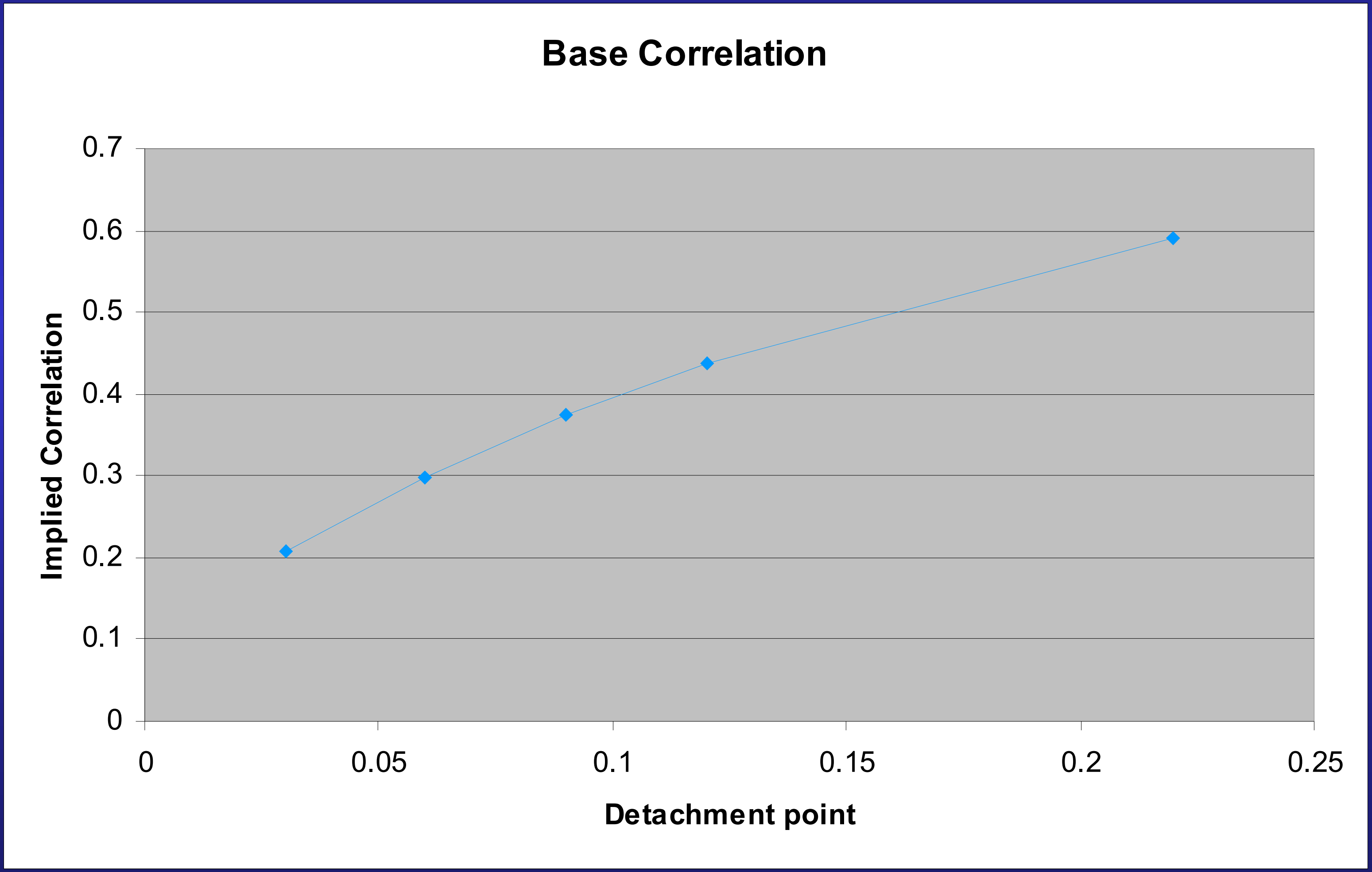
Ways to introduce dependence

- Using a copula: eg Gaussian copula (Li Model), or student-t copula
  - Effectively means correlating defaults according to some easier to model joint distribution and then mapping to default times in such a way that individual default probabilities are correct
  - In a strict mathematical sense, specifying a copula and individual default probabilities is equivalent to specifying the joint distribution.
- Stochastic credit spreads
  - Advantage of realism
  - Difficult to use
  - Hard to obtain enough correlation

Existence of correlation smiles

- Gaussian copula has become a way of implying correlation.
- Different CDOs with the same names and same maturities are priced in the market with differing correlations.
- This gives rise to “correlation smiles” just as in other markets the use of the Black-Scholes model gives rise to volatility smiles.
- The price of an equity tranche is a decreasing function of correlation, so there is a one-one correspondance between price and correlation for equity tranches. This is called “base correlation.”

Typical base correlation smile



The need for new models

- Liquidly traded indices now exists. For index tranches one can mark to market against observed prices.
- We want to price other products consistently with observed prices.
  - Bespoke baskets
  - CDO-squareds
- Gaussian copula has unappealing features
  - Time inhomogeneous
  - Odd dynamics

What causes dependence?

- In bad times, lots of companies default. In good times, few do.
- Defaults cluster according to economic conditions.
- How can we express this in a model?
  - Background economic driver affecting defaults
  - Two (or N) state model
    - Default probabilities much higher in bad years
  - Main problem is tractability

Stochastic time

- We think of time passing being equivalent to many small shocks arriving each of which has a small probability of making an asset default.
- A good year then has a smaller number of small shocks.
- A bad year has an extra large number of small shocks.
- This introduces correlation by increasing the default probability for all assets in bad years, and decreasing it in good years.

Gamma processes

- Make time evolve according to a Gamma process, $\Gamma(t)$.
- A Gamma process is a strictly increasing process with independent increments.
- Time moves in a large number of small jumps.
- The Gamma process is specified by two parameters determining the mean and variance of time across an interval.
- Variance Gamma model for stock evolution based on the idea that time is stochastic.

Intensity Gamma models

- Time evolves stochastically according to a Gamma process
- Assets default according to Poisson processes but with stochastic time instead of calendar time.
- Dependence introduced by the use of the same Gamma process for all assets.
- If Poisson intensity is c then default probability in the interval s to t, provided non-default up to s in equal to

$$\exp(-c(\Gamma(t)-\Gamma(s))$$

Calibrating Intensity Gamma Models

- We need to calibrate to price single name credit default swaps correctly.
- This is equivalent to matching the cumulative probability of default up to each time.
- If the default intensity is c , the conditional probability of default during an interval (s,t) is equal to

$$\int \exp(-cx) f_{(s,t)}(x)dx$$

if $f_{(s,t)}$ is the density of the gamma distribution across (s,t).

Calibrating Intensity Gamma Models

- Default probability across a time interval is

$$\int \exp(-cx) f_{(s,t)}(x)dx$$

- This is the Laplace transform of the Gamma density and it exists in close form.
- This means that via a simple numerical inversion, a unique piece-wise constant intensity as a function of calendar time will match the default probabilities.
- Note: alternative route parametrize via stochastic time

Monte Carlo Pricing with Intensity Gamma Models

- First draw a path for time.
- Conditional on a time path, the default times are independent.
- Draw each default time individually.
- Then compute discounted pay-off.
- Average over many paths.
- Computational work only slightly greater than for Monte Carlo pricing in a one-factor Gaussian copula model.

Drawing a time path

- Simple approach: Divide time into little steps and draw a Gamma distributed across each
- More subtle approach: use fact that a Gamma process is a collection of small jumps
  - Draw the jump sizes and locations
  - Keep on drawing until the remaining jumps are small.
  - Expression given in Cont and Tankov: for interval [0,T]

$$X_t = \sum_{i=1}^{\infty} \lambda^{-1} \exp(-A_i / \gamma T) V_i 1_{TU_i \le t}$$

  - U_i uniforms, A_i arrival times of Poisson process, λ and γ parameters of gamma process, independent V_i exponential r.v.s

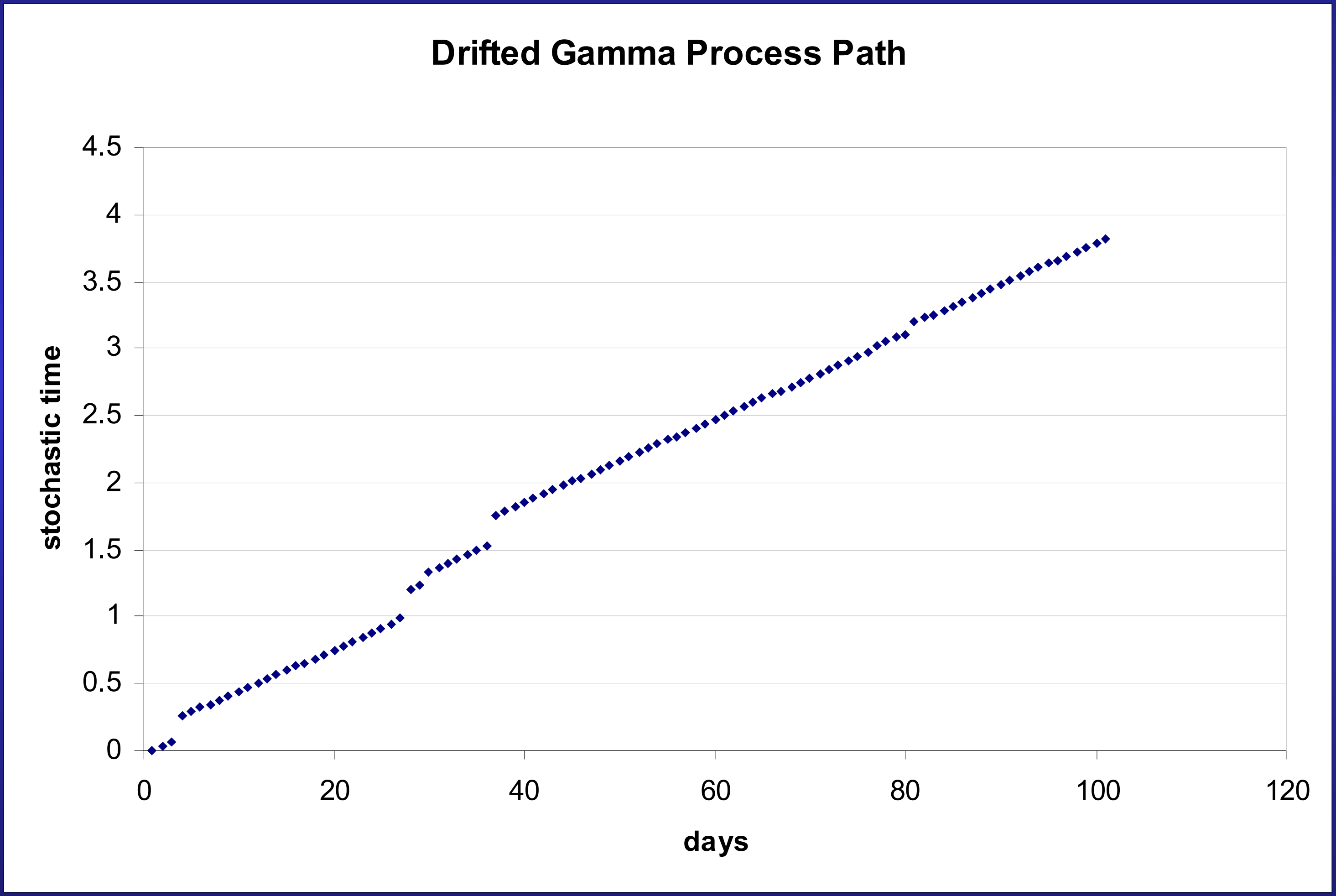
Intensity Gamma Smile

- A gamma process is determined by two parameters equivalent to the mean and variance rate.
- However, calibration against the vanilla essentially fixes the mean rate so we only have one parameter to play with.
- Base correlation smiles are too complex to match with one parameter.

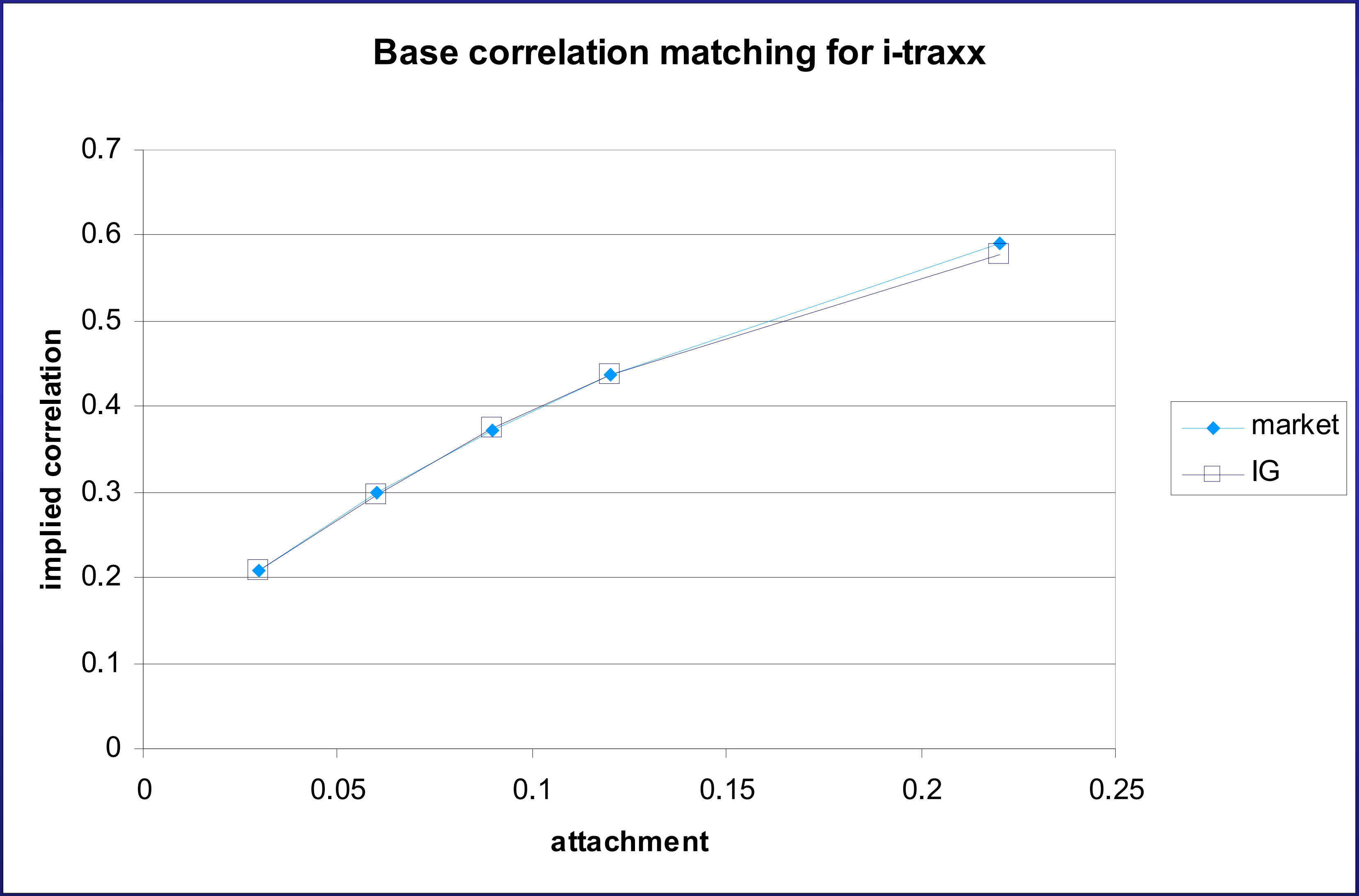
Generalizations of intensity gamma

- We can use any increasing stationary process for stochastic time.
- A sum of two Gamma processes is not a Gamma process so we can use multiple Gamma processes together.
  - Intensities just add so calibration is no harder.
- Use a Gamma process of zero variance i.e. a deterministic time as part of the stochastic time.
- Alpha-stable processes

Typical time path

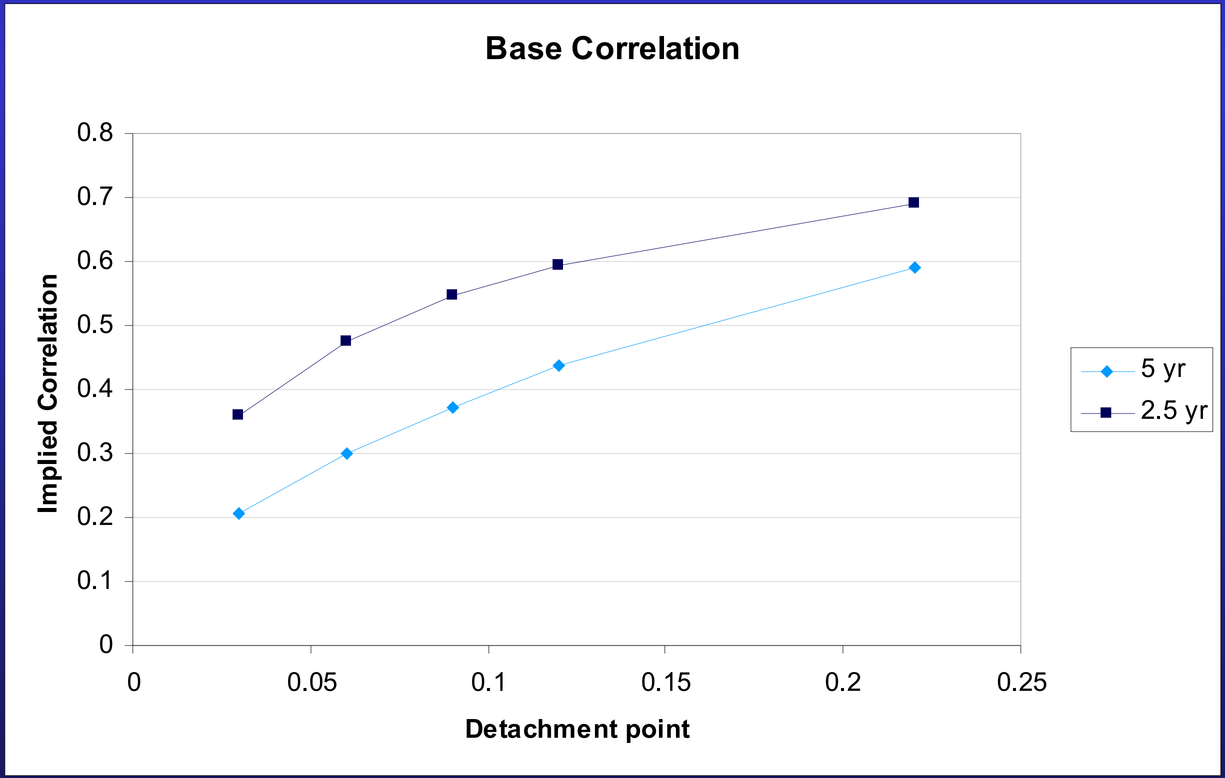


How well do they match the smile?



Implied smiles at different horizons

- Intensity Gamma is time homogeneous so we can use it to imply the base correlation smile for differing horizons.
- The Gaussian copula model is not time homogeneous....



Multi-factor models

- Pairwise dependence for all assets is the same in the model presented.
- Extension of the model
  - Introduce a Gamma process for each sector
  - Introduce a Gamma process for each country

Beyond intensity gamma

- What are the limitations?
  - Deterministic credit spreads
  - Simultaneous defaults
- Could combine stochastic time with stochastic credit spreads cf VGSV or CGMYSV
- Could introduce random time lag after default causing event to remove simultaneity.
- Main issue is tractability.